RA 19

10.6 Subset and permutation examples

- Consider a distribution process a teacher takes to give a set of 4 prizes to 4 out of 10 students in a class.
  - If prizes are identical → use subset
  - If prizes are different → use permutations

General Tip

- If order matters: use permutations
- If order does NOT matter: use subsets

10.7 Counting by Complement

- Counting by complement is a technique for counting the number of elements in a set S that have a property by counting the total number of elements in S and subtracting the number of elements in S that do not have the property.
  - ↳ Can be written using set notation where P is the subset of elements in S that have the property:

$$|P| = |S| - |\bar{P}|$$

Example

- There are 10 kids on math team. There are 6 girls and 4 boys.
  - ↳ How many ways are there to select 2 competitors?
    - ↳ $\binom{10}{2} = 45$
  - ↳ How many ways are there to select 2 girls?
    - ↳ $\binom{6}{2} = 15$
  - ↳ How many ways are there to select 2 competitors so that at least one boy is chosen?
    - ↳ $\binom{10}{2} - \binom{6}{2} = 30$

10.8 Permutations with repetitions

- A permutation with repetition is an ordering of a set of items in which some of the items may be identical to each other.

- Formula: $\frac{n!}{n_1!\, n_2! \cdots n_k!}$

- Example: How many ways to scramble mississippi?
  - ↳ 1st there are $\binom{11}{2}$ choices to place 'p'
  - ↳ then $\binom{9}{4}$ choices to place 'i'
  - ↳ then $\binom{5}{4}$ choices to place 's'
  - ↳ then $\binom{1}{1}$ choices to place 'm'
  - ↳ so $\binom{11}{2}\binom{9}{4}\binom{5}{4}\binom{1}{1} = \frac{11!}{(2!)(4!)(4!)(1!)}$